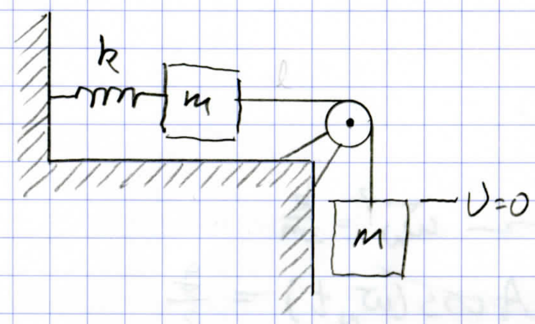
B&D Pr 3.2

TWO EQUAL MASSES ARE CONSTRAINED AS SHOWN. ASSUME A FRICTIONLESS SURFACE & MASSLESS PULLEY. LET x BE THE EXTENSION OF THE SPRING FROM EQUILIBRIUM. DERIVE THE EQUATIONS OF MOTION BY LAGRANGIAN METHODS. SOLVE FOR x AS A FUNCTION OF TIME USING BOUNDARY CONDITIONS: $x(t=0) = \dot{x}(t=0) = 0$



WRITE ENERGIES ($U=0$ WHEN $x=0$)

$$T = \tfrac{1}{2}m\dot{x}^2 + \tfrac{1}{2}m\dot{y}^2$$

$\dot{x}$ HORIZ = $\dot{x}$ VERT

$$T = m\dot{x}^2$$

$$U = -mgx + \tfrac{1}{2}kx^2$$

THE LAGRANGIAN IS

$$L = m\dot{x}^2 + mgx - \tfrac{1}{2}kx^2$$

APPLYING L'S EQUATION

$$\frac{\partial L}{\partial x} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = 0$$

TAKE DERIVATIVES

$$\left.\begin{aligned} \frac{\partial L}{\partial x} &= mg - kx \\ \frac{\partial L}{\partial \dot{x}} &= 2m\dot{x} \\ \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} &= 2m\ddot{x} \end{aligned}\right\} \quad mg - kx - 2m\ddot{x} = 0$$

$$\Rightarrow \ddot{x} + \frac{k}{2m}x - \frac{g}{2} = 0$$

$$\omega_N^2 = \frac{k}{2m}$$

GUESS A SOLUTION:

$$x = A + B\cos(\omega_N t)$$

$$\dot{x} = -\omega_N B\sin(\omega_N t)$$

$$\ddot{x} = -\omega_N^2 B\cos(\omega_N t)$$

APPLY BOUNDARY CONDITIONS

$$x(t=0) = A + B\cos(0) = 0 \Rightarrow A = -B$$

$\dot{x}(t=0) = -\omega_N B\sin(0) = 0$ TELLS US NOTHING! $\longrightarrow$

RE-WRITE x & PUT IT IN THE DE:

$$x(t) = A\left[1 - \cos(\omega_N t)\right]$$

$$\dot{x}(t) = \omega_N A \sin(\omega_N t)$$

$$\ddot{x}(t) = \omega_N^2 A \cos(\omega_N t)$$

FIND A TO MAKE IT A SOLUTION:

$$\ddot{x} + \frac{k}{2m} x = \frac{g}{2} \qquad \omega_N^2 = \frac{k}{2m}$$

$$\Rightarrow \cancel{\omega_N^2 A \cos(\omega_N t)} + \frac{k}{2m} A - \cancel{\omega_N^2 A \cos(\omega_N t)} = \frac{g}{2}$$

$$\frac{kA}{m} = g$$

$$\Rightarrow \boxed{A = \frac{mg}{k}}$$

THUS THE SOLUTION IS

$$\boxed{x(t) = \frac{mg}{k}\left[1 - \cos(\omega_N t)\right]}$$ TA DA!